The volume of Gaussian states by information geometry

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The notion of phase space is often gotten rid of in quantum mechanics because of the non-commutativity of canonical variables. Nevertheless, the phase space can be considered as a common playground for both classical and quantum states when one employs for the latter a description in terms of the so-called quasi-probability distribution functions, such as the Wigner function. Then, one can address the computation of the volume of different classes of states in the phase space framework. The issue of the volume of sets of states is of uppermost importance. It can help in distinguishing classical from quantum states as well as to find separable states within all quantum states. Determining the volume of physical states is also relevant for defining the typical properties of a set of states.

Describing the geometric properties of sets of states is intimately connected with the evaluation of their volumes. The sets of classical and quantum states are both convex sets. In finite dimensional systems, several metrics are introduced in order to compute the volume of physical states. However, when going to infinite dimensional systems, problems arise also from the non-compactness of the support of states. Thus, on the one hand, we have the difficulties in analysing infinite dimensional systems, while on the other hand we still lack a unifying approach for evaluating volumes of classical and quantum states. To deal with these problems, we propose to exploit information geometry.

Actually we exploit methods of information geometry in order to associate a Riemannian manifold to a generic Gaussian system. In such a way, we consider a volume measure as the volume of the manifold associated with a set of states of the system. We start by considering N identical and indistinguishable particles, i.e., bosonic modes, characterized by their positions and momenta, and we assume that a Gaussian pdf with zero mean value describes the whole system state. Such a pdf is characterized by a set of parameters, i.e., the entries of the covariance matrix (depending on their values, we can have various classes of states). Then, thanks to these parameters, to each class of states is associated a statistical model which turns out to be a Riemannian manifold endowed with the well-known Fisher-Rao metric. We are able to overcome the difficulty of an unbounded volume by introducing a regularizing function stemming from energy bounds, which acts as a form of compactification of the support of Gaussian states. We then proceed to consider a different regularizing function which satisfies some nice properties of canonical invariance. Finally, we find the volumes of classical, quantum, and quantum entangled states for two-mode Gaussian systems, showing chains of strict inclusions.